Coordination for a Group of Autonomous Mobile Agents with Multiple Leaders

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Abstract: In this paper, we consider the coordination control of a group of autonomous mobile agents with multiple leaders. Different interconnection topologies are investigated. At first, a necessary and sufficient condition is proved in the case of fixed interconnection topology. Then a sufficient condition is proposed when the interconnection topology is switched. With a simple first-order dynamics model by using the neighborhood rule, both results show that the group behavior of the agents will converge to the polytope formed by the leaders.

Key Words: multi-agent systems, multiple leaders, convex set, polytope.

1 INTRODUCTION

Recent years have seen a large and growing literature concerned with the coordination of a group of autonomous agents, partly due to a broad application of multi-agent systems including flocking/swarming (e.g., [5, 6]), formation control (e.g.,[1, 2]), and sensor networks (e.g., [8]). Leader-following problem is one of the important coordination problems in the studies of multiple mobile agents. Even in some leaderless cases, concepts like "virtual leader" are proposed to study cooperative behaviors [9, 13].

Usually, there is only one leader in the leader-following formulation, though sometimes, the leader may be active with unmeasurable states [4]. However, in some practical situations of formation or foraging, the formulation of multiple leaders may be needed. In [2], Lin *et al.* discussed an interesting model for a group of agents with straight-line formation containing two "edge leaders", where all the agents converge to a uniform distribution on the line segment specified by the two edge leaders. In [3], a simple model was given to simulate foraging and demonstrate that, the larger the group is, the smaller the proportion of "leaders" is needed to guide the group.

Inspired by [2, 3], in this paper we consider the coordination behavior of mobile agents with multiple leaders. By using

a neighborhood rule, we show that a group of agents will converge to the polytope formed by the leaders (that is, the leaders forms the vertex set of the polytope) in two different cases of interconnection topologies associated with the agents and the leaders, and demonstrate that the collective behavior changes as the connectivity of the interconnection between agents and some leader increases through some simulations.

This paper is organized as follows. Section 2 presents the problem formulation for multiple leaders. Then, both fixed interconnection topology and switched topology are considered, and the corresponding coordination behaviors are analyzed in Sections 3 and Section 4. In Section 5, two numerical examples are shown. Finally, concluding remarks are given in Section 6.

2 PROBLEM FORMULATION

Before formulating our problem, we first introduce some basic concepts and notations in graph theory that will be used [10].

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a weighted undirected graph of order n with a set of nodes $\mathcal{V} = \{1, 2, ..., n\}$, set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A = [a_{ij}] \in \mathcal{R}^{n \times n}$ with nonnegative elements. The node indexes belong to a finite index set $\mathcal{I} = \{1, 2, ..., n\}$. An edge is denoted by (i, j), which means node i and node j are adjacent. A path from i to j in $\mathcal{G}$ is a sequence of distinct nodes starting with i and ending with j such that the consecutive nodes are adjacent. If

there is a path between any two nodes of a graph $\mathcal{G}$, then $\mathcal{G}$ is connected, otherwise disconnected. An induced subgraph $\mathcal{X}$ of $\mathcal{G}$ that is maximal, subject to being connected, is called a component of $\mathcal{G}$. The element a_{ij} associated with the edge of the graph is positive, i.e. $a_{ij} > 0 \Leftrightarrow (i,j) \in \mathcal{E}$. Moreover, we assume $a_{ii} = 0$ for all $i \in \mathcal{I}$. The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$.

A diagonal matrix $D = diag\{d_1, ..., d_n\} \in R^{n \times n}$ is a degree matrix of $\mathcal{G}$, whose diagonal elements $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$ for $i = 1, ..., n$. Then the Laplacian of the weighted graph $\mathcal{G}$ is defined as $L = D - A \in R^{n \times n}$. The following result is well-known in algebraic graph theory (e.g. [10]) and establishes a direct relationship between the graph connectivity and its Laplacian.

Lemma 1 *Let $\mathcal{G}$ be a graph on n vertices with Laplacian L. Denote the eigenvalues of L by $\lambda_1(L), \cdots, \lambda_n(L)$ satisfying $\lambda_1(L) \leq \cdots \leq \lambda_n(L)$. Then $\lambda_1(L) = 0$ and $\mathbf{1} = [1, 1, \cdots, 1]^T \in R^n$ is its eigenvector. Moreover, if $\mathcal{G}$ is connected, $\lambda_2 > 0$.*

Next, we introduce some notations in convex analysis [12] and linear algebra [11]. Let $S \subset R^m$. S is said to be convex if $(1 - \gamma)x + \gamma y \in S$ whenever $x \in S, y \in S$ and $0 < \gamma < 1$. A vector sum $\gamma_1 x_1 + \cdots + \gamma_n x_n$ is called a convex combination of $x_1, \cdots, x_n$ if the coefficients γ_i are all nonnegative and $\gamma_1 + \cdots + \gamma_n = 1$. Here, γ_i can be interpreted as proportions. The intersection of all convex sets containing S is the convex hull of S, denoted by $co(S)$. The convex hull of a finite set of points $x_1, \cdots, x_n \in R^m$ is a polytope, denoted by $co\{x_1, \cdots, x_n\}$. If $x_2 - x_1, \cdots, x_n - x_1$ are linearly independent, the set of points $x_1, \cdots, x_n$ is said to be affinely independent. Then the polytope is called an $n - 1$ dimensional simplex and $x_1, \cdots, x_n$ are called the vertices of the simplex.

In this paper, we consider a system consisting of n agents and k leaders, and the interconnection topology among them can easily be described by a simple graph. The purpose of the leaders is to guide the multi-agent behavior. Denote the positions of these static leaders by $x_0^j \in R^m, j = 1, \cdots, k$. The interconnection topology $\mathcal{G}$ associated with n agents can be fixed or variable but may not be connected. If we regard the polytope formed by leaders as a virtual node and if one agent can see a vertex (i.e. some leader) of the polytope, we say that the agent is connected to the virtual node. By "the graph, $\bar{\mathcal{G}}$, of this system is connected", we mean that at least one agent in each component of $\mathcal{G}$ is connected to the virtual node. Then we define a diagonal matrix B to be a leader adjacency matrix associated with $\bar{\mathcal{G}}$ with diagonal elements

b_i ($i \in \mathcal{I}$) such that each b_i is some positive number if agent i is connected to the virtual node and 0 otherwise.

A continuous-time dynamics of n agents is described as follows:

$$\dot{x}_i = u_i, \tag{1}$$

where $x_i \in R^m$ can be the position of agent i and $u_i \in R^m$ its interconnection control inputs for $i = 1, ..., n$. As usual, we propose a neighbor-based feedback control as follows:

$$u_i = \sum_{j \in N_i(\sigma)} a_{ij}(x_j - x_i) + \sum_{q=1}^{k} b_i^q(\sigma)(x_0^q - x_i);$$
$$i = 1, \cdots, n. \tag{2}$$

where nonnegative functions $b_i^q(\sigma(t)) > 0$ if and only if agent i is connected to leader q ($q = 1, \cdots, k$) when $t \in [t_l, t_{l+1})$ for $l = 0, 1, \cdots$, and switching signal $\sigma : [0, \infty) \to \mathcal{P} = \{1, ..., N\}$ ($N \in Z^+$, denotes the total number of all possible digraphs) is a switching signal that determines the interconnection topology $\bar{\mathcal{G}}$. If σ is a constant function, then the corresponding interconnection topology is fixed.

Denote

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in R^{mn}, x_0 = \begin{pmatrix} x_0^1 \\ \vdots \\ x_0^k \end{pmatrix} \in R^{km}.$$

Let $B_\sigma^q = diag\{b_1^q(\sigma(t)), \cdots, b_n^q(\sigma(t))\} \in R^{n \times n}$ be a diagonal matrix with non-negative diagonal entry $b_i^q(\sigma(t))$ for $q = 1, \cdots, k$ and $B_\sigma = [B_\sigma^1, \cdots, B_\sigma^k] \in R^{n \times nk}$. Let $\Xi = \{\xi = (\xi_1, \cdots, \xi_n)^T | \xi_i \in co\{x_0^1, \cdots, x_0^k\}\}$.

Then we rewrite the closed-loop system in a compact form:

$$\dot{x} \in F(x), \tag{3}$$

with a set-valued function

$$F(x) = \{-(H_p \otimes I_m)x + [B_p(I_k \otimes \mathbf{1}_n)] \otimes I_m x_0 | p \in \mathcal{P}\}$$

and

$$H_p = L_p + B_p(\mathbf{1}_k \otimes I_n),$$

where $\otimes$ is the Kronecker product.

Throughout the paper, $R^+ = [0, \infty), ||\cdot||$ denotes the Euclidean norm and $<\cdot, \cdot>$ denotes inner product on R^{mn}. For non-empty $\Xi \subset R^{mn}$, $d_\Xi : R^{mn} \to R^+$ denotes its Euclidean distance function given by $d_\Xi(x) = \frac{1}{2} \inf_{\xi \in \Xi} ||x - \xi||^2$.

The objective of this work is to lead all the agents to enter the region formed by the leaders; namely, x_i ($i = 1, \cdots, n$), for

each agent i, will be contained in a convex hull of x_0^j $(j = 1, \cdots, k)$ as $t \to \infty$. In other words,

$$\lim_{t \to \infty} d_\Xi(x(t)) = 0. \tag{4}$$

In the following sections, the convergence of the system (3) will be studied based on $d_\Xi(x)$, with either fixed interconnection topology or switched interconnection topology.

3 FIXED INTERCONNECTION TOPOLOGY

In this section, we will focus on the convergence analysis of a group of dynamic agents with fixed interconnection topology. In this case, the subscript σ can be dropped.
Then the differential inclusion (3) becomes

$$\dot{x} = -(H \otimes I_m)x + [B(I_k \otimes \mathbf{1}_n)] \otimes I_m x_0. \tag{5}$$

The next lemma shows the relationship between the positive definiteness of a matrix $H = L + B(\mathbf{1}_k \otimes I_n)$ and the connectivity of $\bar{\mathcal{G}}$, which was proved in [4].

Lemma 2 *If graph $\bar{\mathcal{G}}$ is connected, then the symmetric matrix H associated with $\bar{\mathcal{G}}$ is positive definite.*

Then a main result associated with the fixed interconnection topology is given as follows:

Theorem 1 *For the system (5), (4) holds if and only if $\bar{\mathcal{G}}$ is connected,*

Proof: (Sufficiency) Let $\bar{x} = x - [H^{-1}B(I_k \otimes \mathbf{1}_n)] \otimes I_m x_0$. Then, from equation (5), we have

$$\dot{\bar{x}} = -(H \otimes I_m)\bar{x}. \tag{6}$$

For any initial value $\bar{x}(t_0)$ at initial time t_0, the solution of the system (6) is

$$\bar{x}(t) = e^{-(H \otimes I_m)(t-t_0)}\bar{x}(t_0), \ t \geq t_0.$$

From Lemma 2, the eigenvalues of $H \otimes I_m$ are positive, and therefore, $\bar{x}(t) \to 0$; namely, $x \to x^* = [H^{-1}B(I_k \otimes \mathbf{1}_n)] \otimes I_m x_0$, as $t \to \infty$.
Then we only need to prove that each vector $x_i^* \in R^m$ ($i = 1, \cdots, n$) can be represented by a convex combination of $x_0^p \in R^m$ for $p = 1, \cdots, k$. It is equivalent to prove that $[H^{-1}B(I_k \otimes \mathbf{1}_n)] \otimes I_m$ is a row stochastic matrix which is a non-negative matrix and the sum of the entries in every row equals 1.
For H, there exists a positive number α such that $H = \alpha I - M$, where M is a non-negative matrix and $\alpha > \rho(M)$ with $\rho(M) = \max\{\sqrt{\lambda}, \lambda$ is an eigenvalue of $M^T M\}$. In fact,

the eigenvalue $\lambda_i(H) = \alpha - \lambda_i(M) > 0, i = 1, \cdots, n$. Thus,

$$(\alpha I - M)^{-1} = \frac{1}{\alpha}(I + \frac{1}{\alpha}M + \frac{1}{\alpha^2}M^2 + \cdots) \geq 0_{n \times n}$$

or equivalently, $(H^{-1} \otimes I_m)$ is a non-negative matrix, and so is $[H^{-1}B(I_k \otimes \mathbf{1}_n)] \otimes I_m$.
Additionally, since

$$(H \otimes I_m)(\mathbf{1}_n \otimes \mathbf{1}_m) = (\sum_{q=1}^k B^q \mathbf{1}_n) \otimes \mathbf{1}_m,$$

we have

$$(H \otimes I_m)^{-1}(\sum_{q=1}^k B^q \mathbf{1}_n) \otimes \mathbf{1}_m$$
$$= ((H \otimes I_m)^{-1}[B(I_k \otimes \mathbf{1}_n)] \otimes I_m)(\mathbf{1}_k \otimes \mathbf{1}_m)$$
$$= ([H^{-1}B(I_k \otimes \mathbf{1}_n)] \otimes I_m)(\mathbf{1}_k \otimes \mathbf{1}_m) = \mathbf{1}_n \otimes \mathbf{1}_m.$$

Note that $[H^{-1}B(I_k \otimes \mathbf{1}_n)] \otimes I_m$ is a matrix with every row sum equal to 1, which leads to the conclusion.
(Necessity) Suppose that $\bar{\mathcal{G}}$ is not connected. Without loss of generality, we also suppose that there are κ components in $\mathcal{G}$, where some components are connected with the virtual node. We can renumber the nodes of $\mathcal{V}$ such that Laplacian matrix associated with $\mathcal{G}$ takes the following form

$$L = \begin{pmatrix} L_{11} & 0 \\ 0 & L_{22} \end{pmatrix}$$

and, correspondingly,

$$B^q = \begin{pmatrix} B_{11}^q & 0 \\ 0 & 0 \end{pmatrix}$$

where $L_{11} \in R^{s \times s}(0 \leq s < n)$ is the Laplacian associated with those components connected with the virtual node, $B_{11}^q \in R^{s \times s}$ is a nonzero diagonal matrix and 0's denote some appropriate zero matrices.
Let $x = (y_1^T, y_2^T)^T$ with $y_1 \in R^{sm}$; $y_2 \in R^{(n-s)m}$. Then (5) become:

$$\dot{y}_1 = -(H_1 \otimes I_m)y_1 + [B_1(I_k \otimes \mathbf{1}_s)] \otimes I_m x_0,$$
$$\dot{y}_2 = -(L_{22} \otimes I_m)y_2,$$

where $B_1 = [B_{11}^1, \cdots, B_{11}^k]$, $H_1 = L_{11} + B_1(I_k \otimes \mathbf{1}_s)$. From the proof for the sufficient condition, it follows that s agents corresponding to y_1 will approach to the leaders for an arbitrary initial conditions. However, the other agents corresponding to y_2 will stay static or diverge to some distinct locations which can arbitrarily exist and may not belong to the polytope $co\{x_0^1, \cdots, x_0^k\}$. This leads to a contradiction. ∎

Remark 1 *If $k = 1$, then Theorem 1 will be consistent with Theorem 4 in [4].*

4 SWITCHED INTERCONNECTION TOPOL-OGY

In this section, we consider the system (3) associated with switched interconnection topology. A result is given as follows.

Theorem 2 *For the system (3), if $\bar{\mathcal{G}}_p$ $(p \in \mathcal{P})$ is connected, then (4) holds.*

Proof: Since the topology $\bar{\mathcal{G}}_p$ $(p \in \mathcal{P})$ is connected, from Lemma 2, H_p $(p \in \mathcal{P})$ keeps positive definite, and moreover, $[H_p^{-1}B_p(I_k \otimes \mathbf{1}_n)] \otimes I_m$ is a row stochastic matrix. Obviously, $[H_p^{-1}B_p(I_k \otimes \mathbf{1}_n)] \otimes I_m x_0 \in \Xi$, from the differential inclusion (3), we have

$$\dot{x} \in \tilde{F}(x) = \{-(H_p \otimes I_m)(x - \xi) | \xi \in \Xi, p \in \mathcal{P}\}. \quad (7)$$

Consider the Dini derivative of $d_\Xi(x(t))$,

$$D^+ d_\Xi(x(t)) \quad = \inf_{\xi \in \Xi} < x - \xi, \dot{x} > \\ \leq -\lambda_1 d_\Xi(x(t)), \quad (8)$$

where λ_1 is the smallest (positive) eigenvalue of all possible H_p for $p \in \mathcal{P}$. Hence, $d_\Xi(x(t))$ is a decreasing nonnegative function and $d_\Xi(x(t)) = 0$ if and only if $x(t) \in \Xi$, then we have $d_\Xi(x(t)) \to 0$ as $t \to \infty$. ∎

5 NUMERICAL EXAMPLES

In this section, we give numerical simulations to illustrate the coordination of multi-agent systems with multiple leaders. The following two examples are considered (see Fig. 1 and Fig. 2).
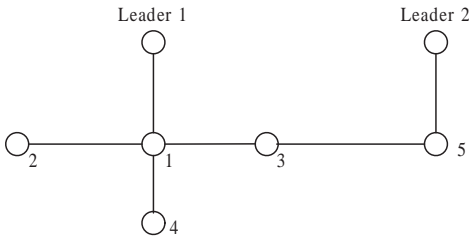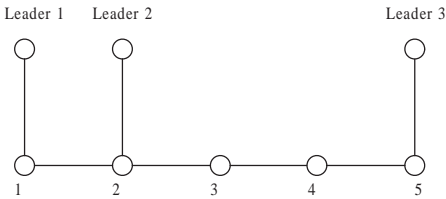

Fig.1. $\bar{\mathcal{G}}_1$ and $\mathcal{G}_1$

Fig.2. $\bar{\mathcal{G}}_2$ and $\mathcal{G}_2$

In the first example, we take $n = 5, k = 2, m = 1, t_0 = 0$ and the initial positions of agents and leaders are given as follows:

$$x_1(0) = 5, \ x_2(0) = 5.5, \ x_3(0) = 6, \ x_4(0) = 7,$$

$$x_5(0) = 6.5, \ x_0^1 = 1, \ x_0^2 = 2.$$

Then the simulation results show that the group of agents will approach to the segment connecting the two leaders in Figs $3 - 6$. In Fig. 4, when the number of agents linked to leader 1 increases, the agent group will move to leader 1 more closely. Moreover, in Fig. 6, when agent 5 is sensed by other agents, then it also moves to leader 1. In Fig. 5, though agent 2 is not connected with other agents but linked to leader 1, then it will reach the locality of leader 1 finally.
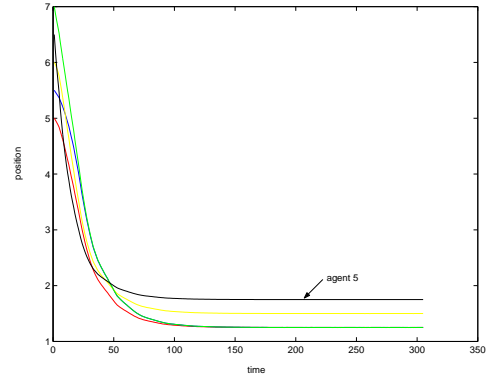

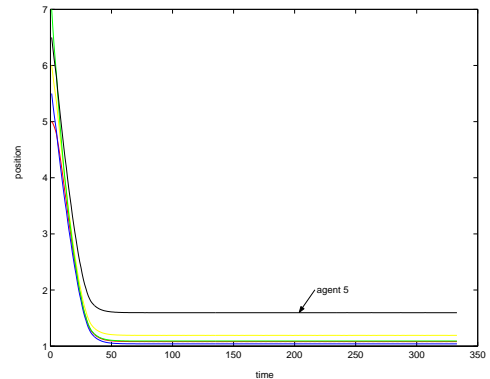Fig.3. Coordination behavior withe the topology shown in Fig. 1

Fig.4. Coordination behavior based on revised Fig. 1, where agents 2,3 and 4 connected with leader 1
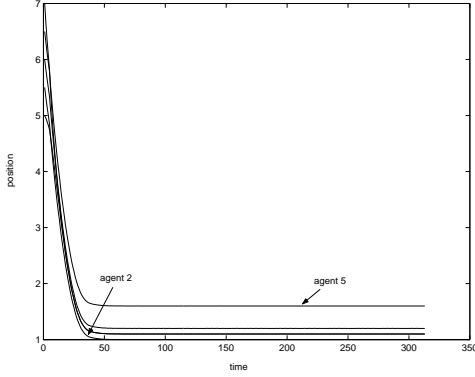
Fig.5. Coordination behavior with the topology associated shown in Fig. 3 with agent 2 disconnected with agents 3 and 4
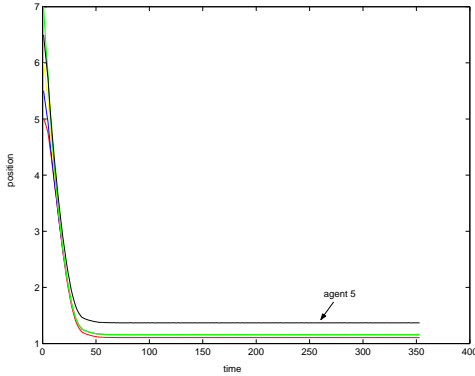

Fig.6. Coordination behavior with the topology associated shown in Fig. 3 with agents 2 and 4 connected with agent 5

In the second simulation example, we take $n = 5, k = 3, m = 2, t_0 = 0$ and the initial positions of agents and leaders are given as follows:

$$x_1(0) = (0,0)^T, \ x_2(0) = (1,0)^T, \ x_3(0) = (2,0)^T,$$

$$x_4(0) = (3,0)^T, \ x_5(0) = (4,0)^T, \ x_0^1 = (1,1)^T,$$

$$x_0^2 = (2,2)^T, \ x_0^3 = (1,2)^T.$$

Here, the agents take a straight-line formation at the beginning.
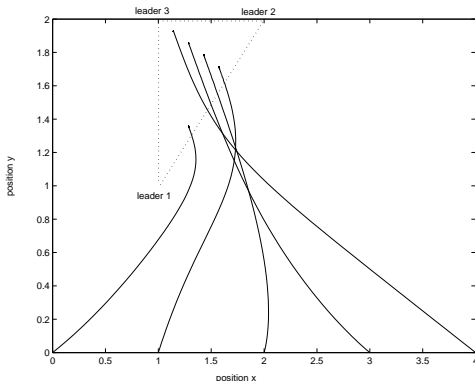

Fig.7. Coordination behavior related to Fig. 2

From Fig. 7, we observe that this group of agents will enter the triangle formed by three leaders. Moreover, during the evolution, agents 2, 3, 4, and 5 still remain straight-line formation.

6 CONCLUSIONS

This paper addressed a coordination problem of a multi-agent system with multiple leaders. This group of agents were shown to approach to the region "contained" by the leaders if the interconnection graph is connected. Two interconnection cases, fixed topology and switched topology, were discussed. Moreover, numerical simulations were given to illustrate the theoretical analysis.

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